

## Dynamics of helping behavior and networks in a small world

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To investigate an effect of social interaction on the bystanders' intervention in emergency situations a rescue model was introduced which includes the effects of the victim's acquaintance with bystanders and those among bystanders from a network perspective. This model reproduces the experimental result that the helping rate (success rate in our model) tends to decrease although the number of bystanders  $k$  increases. And the interaction among homogeneous bystanders results in the emergence of hubs in a helping network. For more realistic consideration it is assumed that the agents are located on a one-dimensional lattice (ring), then the randomness  $p \in [0, 1]$  is introduced: the  $kp$  random bystanders are randomly chosen from a whole population and the  $k-kp$  near bystanders are chosen in the nearest order to the victim. We find that there appears another peak of the network density in the vicinity of  $k=9$  and  $p=0.3$  due to the cooperative and competitive interaction between the near and random bystanders.

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### I. INTRODUCTION

The concepts and methods of statistical physics and non-linear dynamics are applied to investigate the social, economic, and psychological phenomena [1–3]. Among the interesting subjects that have attracted physicists are the opinion dynamics [4,5] including voting process [6,7] and social impact theory [8–13]. Social impact theory stemmed from the bystander effect by which people are less likely to intervene in emergencies when others are present than when they are alone as a result of the inhibitory interaction among bystanders [14,15].

From the laboratory experiments about the emergency situations we can gain an insight into this effect. When tested alone, subjects behaved reasonably and the response rate was high. However, the rate was significantly depressed when they were with other subjects. Subjects with others were unsure of what had happened or thought other people would or could do something. In another experiment subjects who were friends responded faster than those who were strangers. The subjects who had met the victim were significantly faster to report victim's distress than other subjects. And the degree of arousal that bystanders perceive is assumed to be a monotonic positive function of the perceived severity and clarity of the emergency, and bystanders' emotional involvement with the victim [16], which is also to be considered in an abstract way in Sec. II.

In order to investigate the social phenomena as complex systems more precisely we adopt the network point of view by which it means that a social system consists of the interacting agents, where each node and link of the network represent an agent and a relation or interaction between a pair of agents, respectively [17–20]. A number of properties about the real world networks such as social, technological, and biological ones have been revealed and investigated. Two of the main features of the real world networks are the small

world effect and the high clustering to be considered in this paper by introducing a randomness  $p$ , the fraction of the randomly chosen bystanders to the  $k$  bystanders per accident, which plays a similar role to that of the Watts-Strogatz model [21]. The original dimensionless model for the bystander effect is extended to the more realistic and general one in Sec. III.

### II. RESCUE MODEL

Recently in order to investigate an effect of social interaction on the bystanders' intervention in emergency situations a rescue model (RM) was introduced [22]. The model includes the effects of the victim's acquaintance with bystanders and those among bystanders. The RM focuses on the relations between agents rather than on agents themselves, so defined is a relation spin between two agents as  $a_{ij}$  whose value is 1 if agent  $i$  (agent  $j$ ) has succeeded in rescuing agent  $j$  (agent  $i$ ), and 0 otherwise.  $a_{ij}$  is symmetric and can be interpreted as an element of adjacency matrix of helping network. Each agent  $i$  has its intervention threshold  $c_i$  over which that agent can try to intervene in an emergency situation.

At each time step an accident happens which consists of the degree of the clarity or severity of accident represented as a random number  $q_v$  uniformly drawn from  $[0, 1]$ , a randomly chosen victim  $v$  and  $k$  bystanders which are also randomly chosen from a population.  $N_v$  denotes the set of  $k$  bystanders. For each bystander  $i$  the degree of willingness to intervene  $x_{vi}$  is calculated:

$$x_{vi}(t) = q_v + \alpha a_{vi}(t) + \beta \sum_{j \in N_v, j \neq i} [2a_{ij}(t) - 1] - c_i. \quad (1)$$

Only one bystander  $i \in N_v$  with the largest value  $x_{vi}$  can intervene per accident, which can be called the intervener selection rule. If we assume that the response speed of bystander  $i$  is exponential in  $x_{vi}$ , the selection of the bystander with the largest  $x_{vi}$  is justified. Additionally, once one bystander intervenes, the pressures on the others will disappear.

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Then the adjacency matrix is updated as the following:

$$a_{vi}(t+1) = \theta(x_{vi}(t)) \quad (2)$$

where  $\theta(x)$  is a Heaviside step function. If  $x_{vi} \geq 0$ , the rescue succeeds and then for the bystander  $i$  who intervened, the  $a_{vi}$  gets the new value of 1. In case of  $x_{vi} < 0$  the rescue fails and then the  $a_{vi}$  gets the new value of zero.  $\alpha$  represents the degree of victim's acquaintance with bystander, so it can be called an *acquaintance strength*. The third term of  $x_{vi}$  is related to the interaction among bystanders.  $2a_{ij}-1$  gives 1 if one bystander has succeeded in rescuing the other or  $-1$  otherwise. There does not exist any neutral relation here.  $\beta$  is used to tune the strength of coupling so can be called a *coupling strength*. Among them the main control parameter is the number of bystanders  $k$ . As observables we adopt the *network density* [17] (helping rate in Ref. [22]) and the *success rate*, respectively:

$$a_k(t) = \frac{2}{N(N-1)} \sum_{i<j} a_{ij}(t), \quad (3)$$

$$s_k = \frac{1}{T} \sum_{t=0}^{T-1} \theta(x_{vi}(t)). \quad (4)$$

In other words the success rate is defined as the number of successful interventions divided by the total number of interventions. Although the network density can be regarded as a kind of helping rate, the success rate is closer to the helping rate defined in the experimental studies [15] in a sense that the intervention may either succeed or fail without changing the network density. We fix  $c_i \equiv c = 0.25$  for all  $i$  according to the experimental result [14] that 70–75 % of isolated subjects intervened and  $c$  does not change through this paper, which means we consider a population composed of homogeneous nonadaptive agents. Finally, the initial conditions are  $a_{ij} = 0$  for all pairs.

At first let us consider the case without the coupling effect among bystanders, i.e.,  $\beta = 0$ . Generally, an equation for the network density can be written as [22]

$$\frac{da_k(t)}{dt} = W_{0 \rightarrow 1} - W_{1 \rightarrow 0}, \quad (5)$$

where

$$W_{0 \rightarrow 1} = (1-c)[1-a_k(t)]^k,$$

$$W_{1 \rightarrow 0} = (c-\alpha)\{1-[1-a_k(t)]^k\}.$$

$W_{0 \rightarrow 1}$  denotes the probability of creating a new link between the victim and the bystander and  $W_{1 \rightarrow 0}$  does that of eliminating the existing link between them. The stationarity condition for  $a_k$  yields

$$a_k = 1 - \left( \frac{c-\alpha}{1-\alpha} \right)^{1/k}, \quad (6)$$

which says  $a_k$  is a monotonically decreasing function of  $k$ . In the numerical simulations  $a_k(t)$  fluctuates around  $a_k$  since the links are added or removed with finite probabilities  $1-c$  and  $c-\alpha$ , respectively. As  $k$  increases, so does the probability

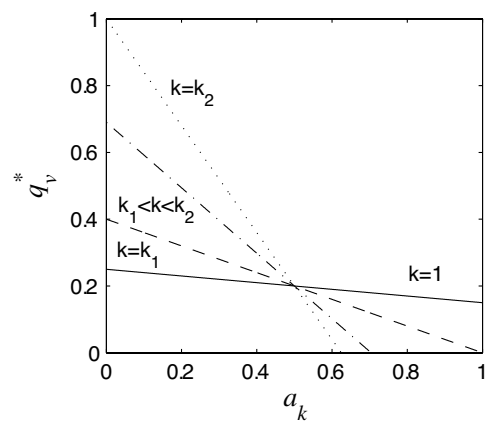


FIG. 1. The diagram for the explanation of the existence of two transition points  $k_1$  and  $k_2$ , where  $k_1 = \frac{c-\alpha}{\beta} + 1$  and  $k_2 = \frac{1-c}{\beta} + 1$  from the mean-field approximation Eq. (9). Here  $\alpha = 0.1$ ,  $\beta = 0.01$ , and  $c = 0.25$  are used.

that two connected agents, one as a victim and the other as a bystander, get involved in an accident again. According to the intervener selection rule one of the bystanders connected with the victim must intervene and thus there is no reason for the increase in  $a_k$  according to  $k$ . Consequently the helping network gets sparse with the number of bystanders.

An equivalent of the success rate defined in Eq. (4) is given by

$$s_k = W_{0 \rightarrow 1} + W_{1 \rightarrow 1} = W_{0 \rightarrow 1} - W_{1 \rightarrow 0} + 1 - (1-a_k)^k = \frac{1-c}{1-\alpha}, \quad (7)$$

where we used the stationary solution for  $a_k$  in Eq. (6).  $s_k$  turns out to be independent of  $k$  and of the network density too. In fact, for the sparser network each link should bear the more burden on the intervention to ensure the success rate constant of  $k$ . From a viewpoint of the uncertainty of a victim's receiving help from the bystanders  $a_k$  corresponds to the cost that the victim should pay to minimize the uncertainty.

If the coupling effect among bystanders is taken into account, then from the definition of  $x_{vi}$  the condition for the successful intervention can be obtained by a mean-field approximation, i.e., the substitution of  $a_k$  for each  $a_{ij}$ :

$$x_{vi} = q_v + \alpha a_k + \beta(k-1)(2a_k - 1) - c \geq 0, \quad (8)$$

or

$$q_v \geq -[\alpha + 2\beta(k-1)]a_k + \beta(k-1) + c \equiv q_v^*. \quad (9)$$

At any time step, when given  $a_k$  the success rate corresponds to  $1 - q_v^*$ . In the case with  $\beta > 0$  there appear two transition points,  $k_1 = \frac{c-\alpha}{\beta} + 1$  and  $k_2 = \frac{1-c}{\beta} + 1$  (see Fig. 1). At  $k = k_1$  for any accident the rescue succeeds,  $s_k = 1$ , if and only if  $a_k = 1$  while at  $k = k_2$  for any accident the rescue fails,  $s_k = 0$ , if and only if  $a_k = 0$ . In the range of  $k_1 \leq k < k_2$ , it is evident that  $s_k \approx 1$ ,  $q_v^* \approx 0$  for  $a_k \geq \frac{1}{2} + \frac{c-\alpha/2}{\alpha+2\beta(k-1)}$ . Once  $c$  is larger than  $\alpha/2$ , then the helping network is so dense that the probability that the bystander who has not been connected with the victim

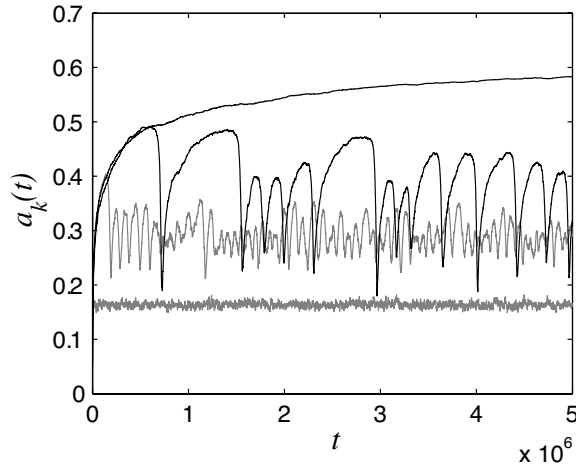


FIG. 2. The numerical results of  $a_k(t)$  for  $k=9$  (lower gray line), 16 (upper gray line), 17 (lower black line), and 18 (upper black line), respectively.  $a_9(t)$  fluctuates around some value.  $a_{16}(t)$  and  $a_{17}(t)$  repeat the slow saturations punctuated by the following abrupt declines.  $a_{18}(t)$  increases monotonically but very slowly and finally approaches 1 as expected in Eq. (11). Here  $N=100$ ,  $\alpha=0.1$ ,  $\beta=0.01$ , and  $c=0.25$  are used.

intervenes is extremely low, so is the possibility of creating a new link. One can expect that  $a_k(t)$  increases since  $s_k \approx 1$ , but very slowly since the network is sufficiently dense.

Given  $W_{1 \rightarrow 0} = 0$  we can calculate the time evolution of  $a_k(t)$  by considering only the  $W_{0 \rightarrow 1}$ . In case that the victim is not connected with any of bystanders, if we assume that at least one bystander is connected with all other bystanders, then for  $k_1 \leq k < k_2$ ,

$$\begin{aligned} \frac{da_k(t)}{dt} &= W_{0 \rightarrow 1} \\ &= [1 - c - \beta(k-1)][1 - a_k(t)]^k \\ &= \beta(k_2 - k)[1 - a_k(t)]^k. \end{aligned} \quad (10)$$

Taking  $a_k(t=0)=0$  as an initial condition yields

$$a_k(t) = 1 - [\beta(k_2 - k)(k-1)t + 1]^{-1/(k-1)}. \quad (11)$$

Therefore  $a_k(t \rightarrow \infty) = 1$  for  $k_1 \leq k < k_2$ . This solution represents the monotonically increasing behavior of the network density with time step and the  $k$  dependence as well.

The time series of  $a_k(t)$  shown in Fig. 2 verify the above arguments except that the transition occurs at  $k=18$  larger than  $k_1$  expected by the mean-field approximation because of the finite size effect. One can see from the Fig. 3 that as the system size increases, the transition point approaches  $k_1=16$ . Additionally  $a_k(t)$  exhibits the punctuated equilibrium-type behaviors at  $k$  slightly smaller than the transition point  $k_1$ , which will be revised in relation to the network viewpoint.

Figure 3 shows the numerical results for  $s_k$  and  $a_k$ , both of which decrease until  $k$  reaches 9–12. This tendency can be interpreted as the bystander effect in that the bystanders are less likely to intervene in emergencies (succeed in rescuing the victim in our model) when others are present than when

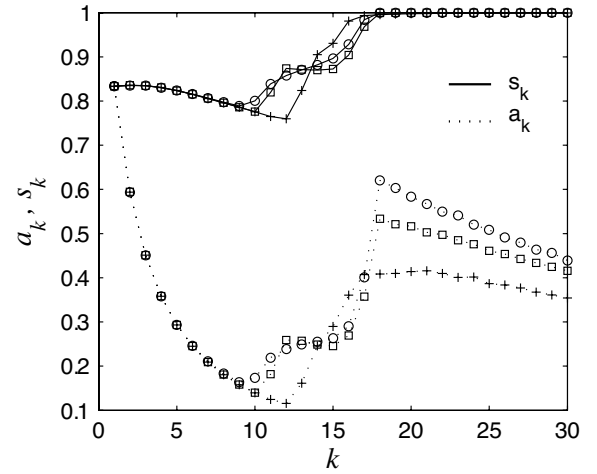


FIG. 3. The numerical results of the network densities and success rates for  $N=100$  (circles),  $N=500$  (squares) and  $N=1000$  (plus signs), respectively. Each data point of  $s_k$  is obtained by averaging over last  $5 \times 10^6$  time steps of entire  $1.5 \times 10^7$  ( $5 \times 10^7$ ) time steps for  $N=100$  (for  $N=500, 1000$ ). And for  $a_k$  the time span is the same as  $s_k$  but each point is averaged over every  $10^3$  time steps. For  $k \geq k_1$   $a_k(t)$  increases monotonically as expected in Eq. (11), however, the points are obtained for finite time steps. Here  $\alpha=0.1$ ,  $\beta=0.01$ , and  $c=0.25$  are used.

they are alone. Contrary to the case with  $\beta=0$  the decreasing  $a_k$  according to  $k$  has an additional negative effect on the coupling among bystanders due to the positive  $\beta$ , thus lowers the degrees of willingness  $x_{vi}$  in Eq. (8) and consequently  $s_k$ . However,  $s_k$  and  $a_k$  are getting large as  $k$  approaches  $k_1$  because of the excitatory coupling among bystanders.

Next, let us focus on the effects of the acquaintance strength  $\alpha$  and the coupling strength  $\beta$  on the structure of helping networks. If  $\alpha=\beta=0$ , since the degrees of willingness for all bystanders are the same as  $x_{vi}=q_v-c$ , the helping network shows a completely random structure. If we consider the acquaintance effect, i.e.,  $\alpha>0$ , the probability that two connected agents get involved in an accident again increases. Therefore  $\alpha$  has a “fixation” effect on the helping network. If the coupling among bystanders is taken into account, i.e.,  $\beta>0$ , the probability that the bystander connected with more other bystanders is more likely to intervene in an emergency, thus  $\beta$  has an effect of “preferential attachment (PA)” on the helping network. As a result of the PA the heterogeneous hubs and hierarchical structures emerge from the homogeneous nonadaptive population. The PA has been investigated and summarized in Refs. [18–20].

In addition, interestingly the above punctuated equilibrium-type behaviors of  $a_k(t)$  in Fig. 2 accompany the rises and falls of hubs when they undergo the slow saturations punctuated by the abrupt declines. The nontrivial total collapse of helping networks can result from the chain reaction between the effect of cutting links due to the rescue failure and that of the rescue failure due to the increasing negative interaction among bystanders. This phenomenon is very different from those of other cases in which once one agent becomes a hub, it lasts forever.

### III. RESCUE MODEL IN A SMALL WORLD

In the previous section we ignored the spatial property of the system which does matter in realities. For more realistic consideration the randomness  $p \in [0, 1]$  is introduced: when assumed that the agents are located on a one-dimensional periodic lattice (ring), the  $k_r \equiv kp$  among  $k$  bystanders are randomly chosen from a whole population, which can be called the *random bystanders*, and the  $k_n \equiv k - k_r$  bystanders are chosen in the order in which they are nearest to the victim in the Euclidean space, which can be called the *near bystanders*. The near bystanders are to the local neighborhoods what the random ones are to the travelers from other places and so on. The randomness  $p$  makes the long-range interaction possible and plays the similar role in our model to the randomness defined as a control parameter of Watts-Strogatz small world networks [21].

#### A. Agents in the one-dimensional world

Let us first consider the case with  $p=0$  which means that all the bystanders are the near ones. In case of even  $k$ , one half of bystanders are left to the victim and the other half are right to the victim. In case of odd  $k$ ,  $k-1$  bystanders are chosen as for the case of even  $k$  except that the side of the last (farthest) bystander is chosen randomly, that is, left or right to the victim. We define a new observable  $y_k$  as the following:

$$y_k(t) = \frac{1}{\lceil k/2 \rceil N} \sum_{i < j} a_{ij}(t), \quad (12)$$

where  $\lceil x \rceil$  is a ceiling function and the denominator is the maximum number of links limited by the locality of interaction. By the definition of  $a_k$ ,

$$a_{k,p=0} = \frac{2}{N(N-1)} \sum_{i < j} a_{ij} = \frac{2\lceil k/2 \rceil}{N-1} y_k. \quad (13)$$

In the case with  $\beta=0$ , since Eqs. (5)–(7) for the case without locality, i.e., for  $a_{k,p=1}$ , are valid for  $y_k$ , it is natural to regard  $y_k$  as  $a_{k,p=1}$ . Thus for small values of  $k$  the network for the case limited by locality becomes much sparser than that for the case without locality. Interestingly  $s_{k,p=0}$  turns out to be independent of  $k$  again, precisely  $s_{k,p=0} = s_{k,p=1} = \frac{1-c}{1-\alpha}$ . Similar to the reason for the  $k$  independence of  $s_k$ , in one-dimensional rescue model the probability that two connected agents get involved in an accident again is very high, thus the helping network gets sparse and as a result each link bears more burden on the intervention.

In the case with  $\beta > 0$  the helping networks in the one-dimensional world consist of a few hubs induced by the PA effect and their peripheries. The number of hubs amounts to about  $N/k$  and the number of peripheries per hub does to about  $k$  as shown in Fig. 4. Once the degrees of any agents become larger than those of others by chance, they eventually grow to the hubs and intervene in emergencies involved with their own peripheries and vice versa, which forms some kind of helping communities. In addition, although the helping network in Fig. 4(b) does not show the scale-free behav-

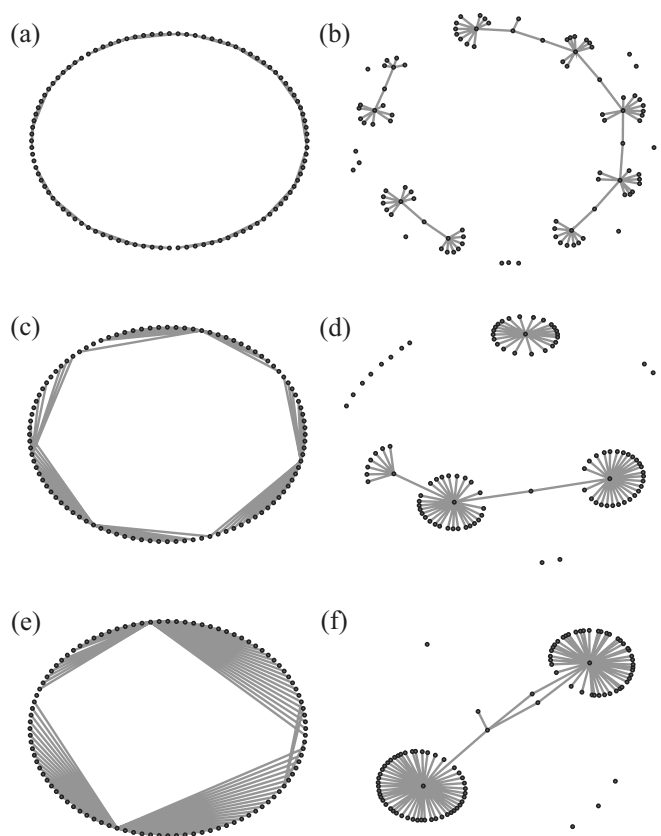


FIG. 4. The numerical results of the helping networks in the one-dimensional world for various  $k$ . The helping network for  $k=11$  is drawn in a circular style (a) and redrawn in (b). (c) and (d) are for the case with  $k=31$  and (e) and (f) are for the case with  $k=55$ , respectively. The hubs emerge from the homogeneous non-adaptive population. Here  $N=100$ ,  $\alpha=0.1$ ,  $\beta=0.01$ , and  $c=0.25$  are used and the networks have been produced with the PAJEK software.

ior of degree distribution its backbone structure bears some resemblance to that of the structured scale-free network [23] (see Fig. 8 in Ref. [24] for comparison).

#### B. Agents in the small world

The network densities and the success rates are scanned for the entire ranges of the number of bystanders  $k$  and the randomness  $p$ . When  $\beta=0$  the numerical results depicted in Fig. 5 show the trivial behaviors. For each  $k$ , according to  $p$  the network density  $a_{k,p}$  leaps from  $a_{k,p=0}$  to about  $a_{k,p=1}$  as soon as at least one random bystander appears, where  $kp_c = 1$  or  $p_c = 1/k$ . For the values of  $p \geq p_c$  the network densities rarely change regardless of  $p$ , which implies that what is relevant is only whether the interaction is local or not and the other factors do not matter. The uncertainty of receiving help is maximized at  $k=1$  and  $p=1$ , where there is only one bystander chosen completely randomly per accident. Therefore the network density should be maximized to ensure the success rate. As seen in Fig. 5(b),  $s_{k,p}$  is independent of  $k$  as well as of  $p$  since the coupling effect among bystanders is not taken into account.

If the coupling effect among bystanders is considered then an interesting phenomenon is observed in Fig. 6, that is,

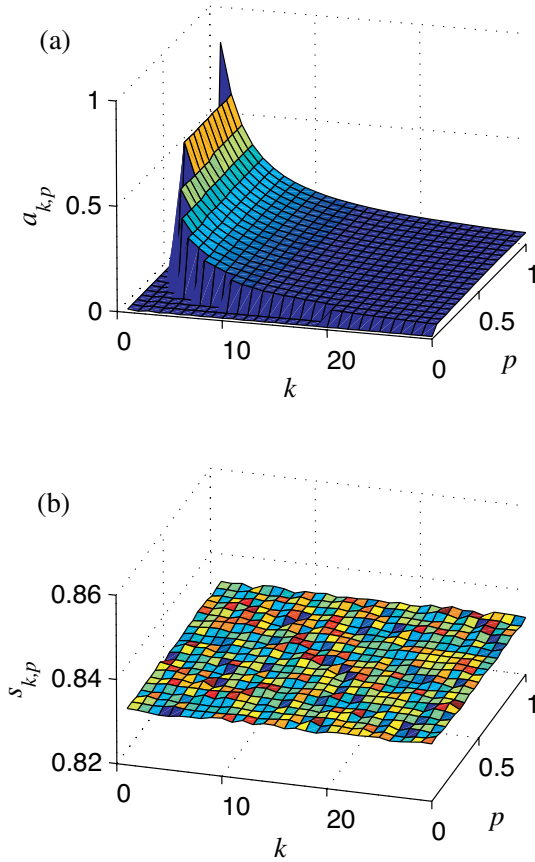


FIG. 5. (Color online) The numerical results of the network density and the success rate for  $1 \leq k \leq 30$  and  $0 \leq p \leq 1$  when  $\beta=0$ . Here  $N=100$ ,  $\alpha=0.1$ , and  $c=0.25$  are used.

there appears another peak of  $a_{k,p}$  and  $s_{k,p}$  in the vicinity of  $k=9$  and  $p=0.3$ . To understand the new peak in the intermediate range of both  $k$  and  $p$  we focus on a cooperative or competitive interaction between two groups; group of near bystanders and that of random ones. Based on the relevance of positive  $\beta$  we conjecture that the clustered structure of near bystanders is essential to enhance the random bystanders' intervention and their possibility of success so that the overall network density dominated by the nonlocal links can grow to a large value. When  $k$  fixed such as 9, for small  $p$  (large  $k_n$ ) local interactions among clustered near bystanders dominate nonlocal interactions among random ones and those across near and random ones, where the locality restrains the network density from getting large. For intermediate  $p$  and  $k_n$  the random bystanders are benefited from the clustered near ones by making use of the existing links across near ones and random ones near to the near ones and then the range of interaction is expanded to the whole system after all. Consequently the network density becomes large. Finally, for large  $p$  (small  $k_n$ ) the near bystanders rarely cluster so that the random ones cannot be benefited from the clustering of near ones, hence the network density will converge to  $a_{k,p=1}$  rather than increase.

To verify the above conjecture, at first two network densities are newly introduced:

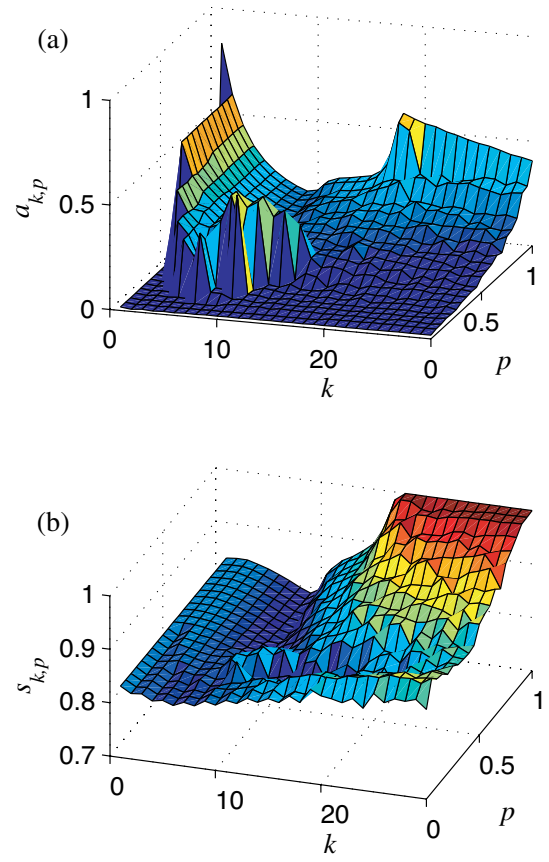


FIG. 6. (Color online) The numerical results of the network density and the success rate for  $1 \leq k \leq 30$  and  $0 \leq p \leq 1$  when  $\beta=0.01$ . It is found that there appears another peak of  $a_{k,p}$  and  $s_{k,p}$  in the vicinity of  $k=9$  and  $p=0.3$ . Here  $N=100$ ,  $\alpha=0.1$ , and  $c=0.25$  are used.

$$a_{k,p}^{(n)}(t) = \frac{1}{Nk_n} \sum_{d(i,j) \leq k_n} a_{ij}(t), \quad (14)$$

$$a_{k,p}^{(r)}(t) = \frac{1}{N(N-1)/2 - Nk_n} \sum_{d(i,j) > k_n} a_{ij}(t), \quad (15)$$

where  $d(i,j)$  gives the shorter distance on the ring between agent  $i$  and  $j$  and the superscripts  $n$  and  $r$  represent the near and random bystanders, respectively.  $Nk_n$  in Eqs. (14) and (15) is the maximum number of links among near bystanders. From the extensive numerical simulations it is found that for the values of  $k$  and  $p$  other than the new peak region and the growth region of  $k_1 \leq k < k_2$  and  $p$  near to 1, both  $a^{(n)}(t)$  and  $a^{(r)}(t)$  fluctuate around some values. On the other hand, for the new peak region, precisely at  $k=9$  and  $p=0.3$ , in Fig. 7(a)  $a^{(n)}(t)$  jumps to about 0.15 very quickly and fluctuates around that value for a while until  $a^{(r)}(t)$  grows exponentially to exceed  $a^{(n)}(t)$ , which is called the intersection point.  $a^{(r)}(t)$  increases fast then saturates while  $a^{(n)}(t)$  also increases a little. To figure out whether two bystander groups cooperate or compete we calculate the cross correlations between two network densities before and after the intersection point. Before that point the cross correlation is

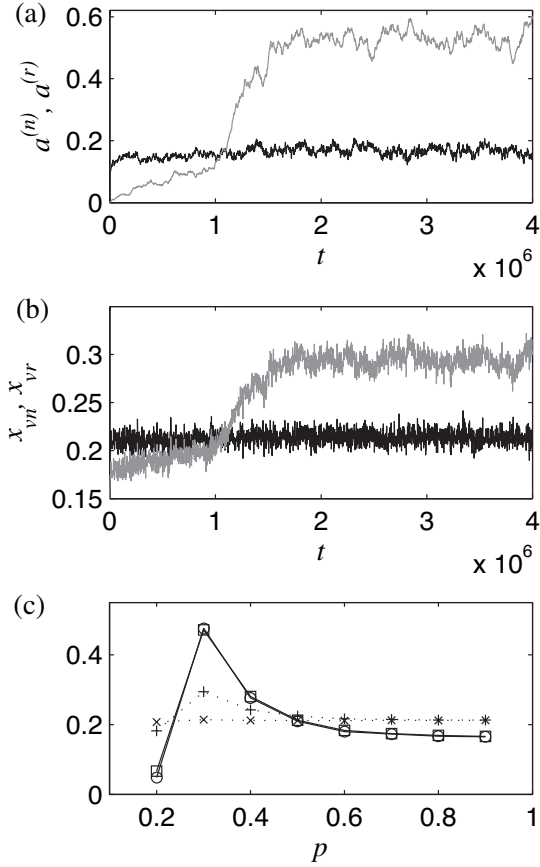


FIG. 7. The numerical results for verifying the importance of clustered near bystanders. (a) Time evolutions of two network densities introduced in Eqs. (14) and (15);  $a^{(n)}(t)$  for near bystanders (black line) and  $a^{(r)}(t)$  for random ones (gray line). (b) Time evolutions of the degrees of willingness;  $x_{vn}(t)$  for near bystanders (black line) and  $x_{vr}(t)$  for random ones (gray line). For (a) and (b) each point is averaged over 4000 time steps. (c) The verification of Eq. (16) with  $k=9$  by numerical simulations;  $x_{vn}$  (crosses),  $x_{vr}$  (plus signs),  $a_{k,p}^{MF}$  (squares), and  $a_{k,p}$  (circles). Each point is averaged over 50 realizations after saturated. Since there is no random bystanders for  $p \leq 1/k$  and there is no near ones for  $p=1$ , in these ranges of  $p$  either  $x_{vr}$  or  $x_{vn}$  cannot be defined. Here  $N=100$ ,  $\alpha=0.1$ ,  $\beta=0.01$ , and  $c=0.25$  are used.

0.5236 while after that point it is  $-0.3565$ . In the early stage the near bystanders help the random ones intervene but once the random ones dominate the near ones, two groups compete for the chance of intervention.

For more specific investigation we calculate the averaged degree of willingness of random bystanders  $x_{vr}(t)$  as a function of time and that of near ones  $x_{vn}(t)$  respectively as shown in Fig. 7(b). Similar to the time evolution of  $a^{(r)}(t)$ ,  $x_{vr}(t)$  increases from a lower value than  $x_{vn}(t)$  then exceeds it at the intersection point while  $x_{vn}(t)$  fluctuates around some value.  $x_{vr}$  larger than  $x_{vn}$  implies the more chance of random bystanders' intervention than that of near ones' intervention. As a result the success rate of random ones ( $0.8531 \pm 0.0101$ ) is also slightly larger than that of near ones ( $0.8212 \pm 0.0221$ ). The difference of two averaged degrees of willingness affects which kind of bystanders are more likely

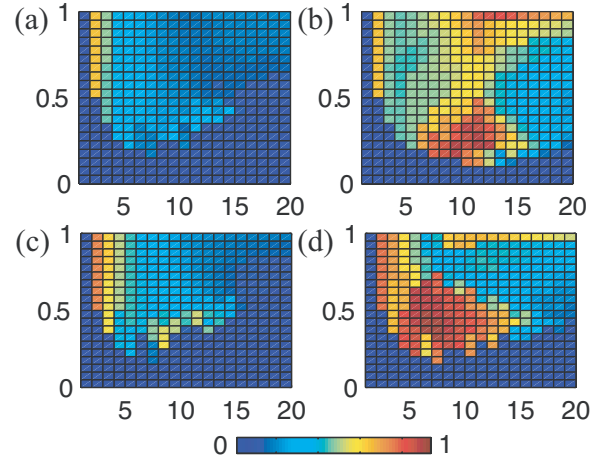


FIG. 8. (Color online) The numerical results of the landscape of network density for  $1 \leq k \leq 20$  (horizontal axis) and  $0 \leq p \leq 1$  (vertical axis), changing from the sharp peak to a plateau according to  $\alpha$  and  $\beta$ : (a)  $\alpha=0.05$ ,  $\beta=0.005$ , (b)  $\alpha=0.05$ ,  $\beta=0.02$ , (c)  $\alpha=0.2$ ,  $\beta=0.005$ , and (d)  $\alpha=0.2$ ,  $\beta=0.02$ . Each point is averaged over ten realizations. Here  $N=100$  and  $c=0.25$  are used.

to intervene and succeed, which affects the network density and the success rate successively. For general  $k$  by mean-field approximation the degrees of willingness are given by

$$x_{vr} = q_v + \alpha a_{k,p} + \beta(k-1)(2a_{k,p} - 1) - c,$$

$$x_{vn} = q_v + \alpha a_{k,p=1} + \beta(k-1)(2a_{k,p=1} - 1) - c,$$

where we have assumed that the probability that a random bystander is connected with any other bystander is the same as  $a_{k,p}$  and for the near bystanders the probability to be connected with any other one is  $y_k = a_{k,p=1}$ . From these the mean-field network density is obtained:

$$a_{k,p}^{MF} = a_{k,p=1} + \frac{x_{vr} - x_{vn}}{\alpha + 2\beta(k-1)}. \quad (16)$$

The numerical results in Fig. 7(c) for Eq. (16) with  $k=9$  indicate that the mean-field approach works.

For the other values of  $k$  what happens according to the randomness  $p$ ? For smaller values of  $k$  and  $p \geq p_c$  the clustering of near bystanders rarely contributes to the random ones' intervention because the network is relatively dense to ensure the success rate for small  $k$ . Therefore it is the locality that is relevant for the results as for the case with  $\beta=0$ . For larger values of  $k$ , especially larger than  $k_1$ , we already know the network density goes to 1 but very slowly when  $p=1$ . Since for small  $p$  the network density converges to some value, there should be a transition line where the clustering cannot exist, i.e.,  $k_n \leq 1$ , equivalently  $p \geq 1 - 1/k \equiv p^*$ . When  $p < p^*$  and  $k \geq k_1$  the locality limits the long-range excitatory interaction among bystanders and at the same time the probability that the victim and the bystander are already connected is relatively high due to large  $k$  so that it is difficult for near bystanders to form an effective cluster and thus to enhance the random ones' intervention too.

Next, as initial conditions we take the random networks whose network density  $a(0)$  varies from 0.05 to 0.5. Only for the small value of  $a(0)$ , precisely 0.05, there appears the tiny peak in the intermediate range of  $k$  and  $p$  while no peak appears in the other cases with larger  $a(0)$ . This is because the initial random structure does not allow any structural change for the clustering of near bystanders. In conclusion, the new peak in an intermediate range of  $k$  and  $p$  has been made sense by introducing the clustering effect of near bystanders.

Finally it is also observed in Fig. 8 that increasing  $\alpha$  and  $\beta$  lead to the overall increase in the height of peak of network density then change the shape of peak from a sharp one to a plateau. Since  $\alpha$  has a fixation effect on the existing victim-bystander link it cannot contribute to the creation of new link but only can lower the possibility of deleting the existing link. On the other hand  $\beta$  in relation to the bystander-bystander links can lead to the creation of new link between victim and bystander and the deletion of existing link depending on the network density. Therefore around the peak increasing  $\beta$  enhances the successful interventions of random bystanders based on the clustered near ones, then raises the network densities, which are also preserved by increasing  $\alpha$ .

#### IV. CONCLUSIONS

In this paper we have studied not only the original rescue model, which was introduced in order to investigate an effect

of social interaction on the bystanders' intervention in emergency situations, but also the rescue model on a small world. The bystander effect has been successfully reproduced from numerical simulations and explained by the mean-field approximation. In general both of the local interaction and the increasing  $k$  reduce the network density since the victim has more chance to get involved in the acquainted bystander. However, it is found that when the coupling effect among bystanders considered there appears another peak of  $a_{k,p}$  and  $s_{k,p}$  in the vicinity of  $k=9$  and  $p=0.3$  for some given parameters, which results from the enhancement of nonlocal interventions based on the clustering effect of near bystanders.

The relation spins  $a_{ij}$  compose the helping networks. The coupling effect represented by positive  $\beta$  induces the emergence of hubs from a homogeneous nonadaptive population. In the original rescue model the rises and falls of hubs have been observed near the transition point  $k_1$  and in one-dimensional world the whole population is divided into a few helping communities, each of which consists of a hub and its peripheries. Although we could not find any real world helping networks to our knowledge, these results give us an insight into the dynamics of helping behavior and networks.

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